



## Dynamics of stretched flexible tubes conveying fluid

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### ABSTRACT

Silicone polymer tubes offer improved resistance to erosion, light in weight, non-toxic and corrosion-resistant, and hence they are used in many pharmaceuticals as well as medical applications. Usually, the evaluation of the fundamental frequency and the mode shape of fluid conveying pipes are vital in the dynamic analysis of tubes, where the flow parameters like fluid velocity, damping ratio as well as the sagging of flexible structures affect the fundamental frequency and stability. Coriolis mass flow meter (CFM) is a precision flow measurement device which gives precise readings of mass flow rates. This device demands an accurate evaluation of the influence of flow parameters like damping ratio and fluid flow velocity on the fundamental frequency of the system. The flexible tubes possess higher damping ratios, which helps in fast dissipation of the undulations caused by fluid–structure interaction and flow-induced vibrations, compared to the metal tubes. However, the higher damping ratios can sometimes result in destabilisation of the system. The novelty of this paper lies in the development of a unique experimental technique to find the impact of the tube pre-stretch on natural frequency and damping ratio, the influence of mass and stiffness participation of flexible tubes conveying fluid. The damping ratio of the silicone tube is examined by experimenting with different flow velocities and pre-stretches. Study reveals that the pre-stretching of tubes initially decreases the damping ratio, and beyond a particular pre-stretch, the damping ratio increases. Also it is found that for highly pre-stretched tubes, the variation of natural frequency upon different fluid flow velocity is negligible.

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### 1. Introduction

The dynamics of pipe conveying fluid has always been one of the intense areas of interest for the researchers nowadays. The pipes conveying fluids show intense vibration due to the fluid–structure interaction resulting from the transfer of energy from and to the fluid. These vibrations are sometimes potentially risky, which makes the study of fluid–structure interaction and flow-induced vibration interesting for practical engineering problems. The analysis of natural frequencies of pipe conveying fluid helps in identifying the safe operating frequencies of pipes. If the natural frequency of pipe meets with the frequency of the structure, the condition called resonance may occur. It results in uncontrolled oscillations and eventually failure of the joints or system. The silicone rubber tube gains its application in medicinal as well as

healthcare tubes for conveying intravenous fluid, respiratory tubing used in ventilator and heart–lung machines and surgical catheters. Silicone is commonly used material in these applications due to the excellent features such as non-reactant to chemicals or medicines, bio-friendly, long life, strength and ease in making. Devices such as Peristaltic pumps and dosing pumps also gain diverse interest in the medical field due to its peculiar characteristics such as accurate quantity control and delivery of fluid. The silicone rubber tube is used in these devices, which offers a long life of delivery tube with minimum device maintenance. The fluid conveying pipes can become statically unstable, which is commonly known as static instability or divergence for a continuous flexible pipe with supported ends conveying fluid.

This instability of tubes is similar to divergence or buckling of a column subjected to an end load. Negative damping can cause exponentially amplifying oscillations like oscillatory instability or flutter of the cantilevered pipe conveying fluid. Moreover, the cantilevered pipes and simply supported pipes have become most

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common in experimental as well as numerical studies due to the simplicity in modelling equations, delivers interesting dynamic behaviour and broader application in the field of fluid conveying pipes. Hence these types of problems have become a real interest for researchers in the area of dynamics and fluid–structure interaction. In the case of a non-conservative system of the fluid conveying flexible pipe subjected to external excitation, the weak damping can sometimes result in the uncontrolled vibrations as well as destabilization of the system. It has been identified that cantilevered pipes conveying fluid can be destabilized by dissipation. The critical velocity of such a system was found to be decreasing, and hence the dissipation destabilizes the system. Hence the studies of pre-tensioning and energy dissipation are highly crucial in the case of flexible pipes conveying fluids. The most common model for damping in elastic structures is the Rayleigh model. The viscoelastic materials can be modeled using Kelvin-Voigt, Hysteretic or structural damping model. For metals and elastomers like rubber, the dissipation of energy will be more alleged for the hysteresis. Hence for such materials, the energy dissipated per cycle can be calculated by taking Young's Modulus as a complex number. However, this representation of Elastic modulus gives a reasonable approximation for lightly damped oscillations, but the results will be erroneous if there is any flow-induced damping or support damping. It is also reported that the hysteretic damping stabilizes the system with a mass ratio less than 0.285 and destabilizes the system for higher values of mass ratios [1].

The dynamics of fluid conveying pipes have been vastly explained by Paidoussis in his two volumes of the book [1,2]. Paidoussis and Issid [3], J Kin et al. [4] and Vandier et al. [5] studied the effects of damping on tubes conveying fluid. They focused on the effect of vortex-induced vibrations, the effect of boundary conditions and tube materials on the damping of tubes. Pei-xin Gao et al. [6], Tsahalis et al. [7] conducted experiments on the horizontal cylinder whereas Chaplin et al. [8], Vandier et al. [9] and Huera et al. [4] conducted experiments on the vertical cylinders conveying fluid. The damping characteristics of viscoelastic tubes and the effect of damping, vibrations, etc on the support stiffness, fluid velocity and pressure are explored by these authors. Vincent O. S. Olunloyo et al. [10] presented an analytical method to understand the vibrational characteristics of a fluid conveying pipe supported on a viscoelastic foundation. They have investigated the appearance of the beat phenomenon and its effect on the fluid-pipe mass ratio, damping ratio, etc. Mahmoud Abdullatif et al. [11] presented the effect of stabilization as well as the destabilization of the conservative systems subjected to external forces. They have found that the given system can exhibit different types of stability transition behavior in various regions of its parameter space. The studies on flexible pipes conveying fluids examining the effect of flow velocity and pre-stretch on natural frequencies and damping are limited. This paper presents an experimental technique to demonstrate the effect of pre-stretching on damping ratio and natural frequency on flexible pipes conveying fluids. Using analytical Rayleigh damping model the influence of mass proportion and stiffness proportion of damping ratio is also presented.

## 2. Theory

### 2.1. Fluid conveying pipes

The equations of motion of a vertically placed pipe can be given in a general form with the effect of gravity to be non-negligible and the flow velocity may be subjected to small perturbations in the radial direction of the pipe. The pipe's material is assumed to be homogeneous and its lateral motions are assumed to be small and the fluid flow is modeled in terms of a plug flow model [1].

$$\left(E^* \frac{\partial}{\partial t} + E\right) I \frac{\partial^4 w}{\partial x^4} + \left\{ MU^2 - \bar{T} + \bar{p}A(1 - 2\vartheta\delta) - \left[ (M + m)g - M \frac{dU}{dt} \right] (L - x) \right\} \frac{\partial^2 w}{\partial x^2} + 2MU \frac{\partial^2 w}{\partial x \partial t} + (M + m)g \frac{\partial w}{\partial x} + c \frac{\partial w}{\partial t} + (M + m) \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

Usually, the majority of the fluid conveying pipelines are isotropic, metal tubes. These pipes are not stretchable; hence they do not need any pre-tensioning. Also, for metal pipes the flow-related perturbations in pressure and velocity along the radial as well as axial directions, sagging, and changes in damping ratio are very small. Hence most of the researchers neglect those effects in the isotropic metal tubes and Eq. (1) simplifies to

$$EI \frac{\partial^4 w}{\partial x^4} + MU^2 \frac{\partial^2 w}{\partial x^2} + 2MU \frac{\partial^2 w}{\partial x \partial t} + (M + m) \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

The first term in equation (2) is the flexural restoring force, the second term is associated with centrifugal force, the third term is associated with Coriolis force and the fourth term is associated with the inertial force of the fluid-filled pipe. The effects of gravity (or sagging), energy dissipation, and tension cannot be neglected in the case of flexible pipe conveying fluid.

### 2.2. Damping of structures

The damping (energy dissipation) of structures is assumed to be viscous in nature and frequency dependent. The most common method to evaluate damping is to solve the equation of motion using modal analysis techniques in which the damping ratios are directly assigned to the modes. Rayleigh damping indicates the damping as a linear combination of the mass and stiffness matrices [12].

The damping matrix C is given by

$$C = \mu M_s + \lambda K_s \quad (3)$$

The Rayleigh damping results in the different damping ratios for different response frequencies according to the equation

$$\xi = \frac{1}{2} \left( \frac{\mu}{\omega} + \lambda \omega \right) \quad (4)$$

The coefficients  $\mu$  and  $\lambda$  can be determined from the specified damping ratios  $\xi_i$  and  $\xi_j$  for the  $i^{\text{th}}$  and  $j^{\text{th}}$  modes respectively as

$$\frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{Bmatrix} \mu \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \xi_i \\ \xi_j \end{Bmatrix} \quad (5)$$

Two orders of the reference frequencies are selected as first and second fundamental frequencies.

### 2.3. Frequency response function (FRF) and coherence

The frequency response function is a measurement function obtained in the Frequency domain used to identify resonant frequencies, damping ratios and mode shapes of a system. It controls the quality of modal estimation parameters, calculated using input and output signals of the system measured in the time domain.

The FRF of fluid conveying pipe can be evaluated using measured input force (known) and the vibrational response (output signal) in the time domain and then transforming to the frequency domain.

The auto power spectrum of the input signal (measured from force transducer) can be found out as,

$$S_{mm}(f) = \frac{1}{T} M * (f) M(f) \quad (6)$$

Cross spectrum of the input signal and output response are evaluated as:

$$S_{mn}(f) = \frac{1}{T} M * (f) N(f) \quad (7)$$

From this frequency response function can be calculated as,

$$H1(f) = \frac{|S_{mn}(f)|}{S_{mm}(f)} \quad (8)$$

The Coherence function is a dimensionless statistical parameter that gives the correlation between the input and output signals. It helps to identify the periodicity as well as any non-linearity in the input or output signal measured in the time domain. A coherence function value of one indicates that the input and output signals have the best correlation between them whereas zero coherence means output signal does not correlate with the input signal. The coherence function can be evaluated as:

$$\gamma_{mn}^2(f) = \frac{|S_{mn}(f)|^2}{S_{mm}(f) S_{nn}(f)} \quad (9)$$

#### 2.4. Damping ratio

The damping ratios for the flexible tube conveying steady fluid under various Pre-stretches can be calculated using the half-power band width technique [13].

$$\eta d = \Omega 22 - - \Omega 12 / 2 \Omega n 2 \quad (10)$$

### 3. Test facility

The test facility consists of a system to analyze the dynamic characteristics of a horizontal tube conveying fluid. The frame was well designed and fabricated to evaluate the first five fundamental frequencies of the fluid conveying tube, which lies between 0 and 100 Hz. The first fundamental frequency of the structure is found as 120 Hz; therefore, the fundamental frequencies of the tube never match with the frequency of structure during the experimentation. Hence the chances of resonance condition are eliminated. The experiments were carried out in a closed-circuit system of recirculating water. The schematic diagram of the test loop is depicted in Fig. 1 and the experimental set up is given in Fig. 2. The modal estimation parameters are optimized [15] and

are given in Table 1. The silicone tube properties and tube dimensions taken for the experiments are given in Table 2. The system comprises of the flexible silicone tube, two water tanks, sensors, actuators and a data acquisition system. The tube is excited with the help of an electromagnetic shaker (Sentek Dynamics MS-200, capacity 200 N, 5 mm peak to peak amplitude) and the response is acquired with the help of Laser Doppler vibrometer.

#### 3.1. Experimental method

The flexible silicone rubber tube experiences sagging due to self-weight; therefore, the silicone tube is initially stretched by a length of 5% of its initial length. The tube is analysed under clamped-clamped boundary condition with the help of two clamps. One clamp is provided with a rack and pinion system so that it can be moved axially for applying pre-stretch in the tube. The tube test section is divided into eight equal divisions, and those points are marked, as shown in Fig. 1. The endpoints numbered as 1 and 9 are fixed. Reflector tapes were glued to the intermediate seven points for reflection of the laser beams. The dynamic responses of the intermediate seven points are picked up with the help of the laser Doppler vibrometer. The stinger rod of the shaker is attached to the tube via force transducer (Dytran, 22.5 mV/N) which is glued at point 5 of the tube using a strong adhesive.

To ensure a constant flow of water through the tube two storage tanks are used. The water level inside the storage tank is kept constant using float valves. The flow-induced turbulence inside the storage tank is minimized using interconnecting tubes and a constant hydrostatic head is maintained. The silicone tube is excited using a sine sweep signal of frequency ranging from 0 to 100 Hz. This sine wave signal is generated using an arbitrary function generator (Tektronix AFG3022B). The generated signal is amplified using the power amplifier (Sentek Dynamics LA 300, rated output-300 VA). The roving output method was adopted for measuring the response of the structure, in which, the position of the shaker was kept fixed and the vibrometer head was moved from point to point. Flow velocity is measured using a transit-time type ultrasonic flow probe (Portaflow 300, Micronics Ltd, velocity range of 0.05 m/s to 15 m/s) mounted on the inlet pipe. The dynamic response of the tube was measured using a laser Doppler vibrometer (Polytec IVS400). The input signal from the force transducer and the output signal from the vibrometer were collected using

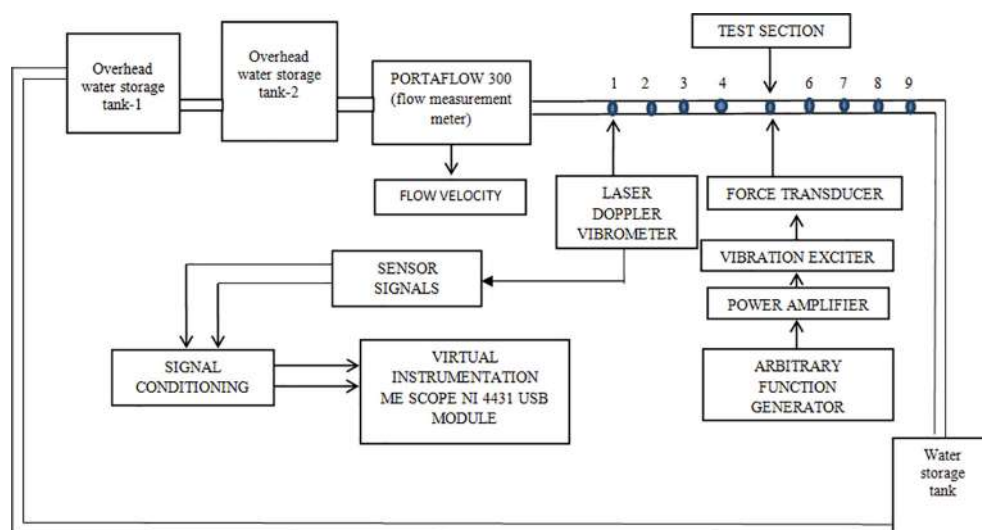


Fig. 1. Layout of the experimentation used for the study of dynamic characteristics of the flexible tube.



Fig. 2. Experimental setup used for the study of dynamic characteristics of the flexible tube.

Table 1  
Modal estimation parameters.

TIME	
Number of samples	8192
Time resolution (sec)	0.0008
FREQUENCY	
Number of samples	4096
Frequency resolution (Hz)	0.153
No of averages	50
WAVEFORM	
Application time (sec)	7
Pause time(sec)	2

Table 2  
Tube dimensions.

Tube material	Silicone rubber
Tube dimensions	Inner diameter = 0.006 m Outer diameter = 0.009 m
Tube material properties	Young's modulus, $E = 5 \text{ Mpa}$ Density, $\rho = 1100 \text{ Kg/m}^3$
Working fluid	Water, $\rho_f = 1000 \text{ Kg/m}^3$
Boundary conditions	Both ends clamped

the DSA. The post-processing of the acquired data is done using ME' scope VES analysis software.

## 4. Results and discussion

### 4.1. Effect of Pre-stretch on the natural frequency of the tube.

In order to find the axial load to be applied for pre-stretching of the silicone tube, a tension test was conducted in computerized UTM. The Load Vs Crosshead travel of silicone polymer tube is shown in Fig. 3. It shows almost a linear trend for about 150 mm of elongation. The approximate loading for 150 mm crosshead travel is 120 N. From the curve the correlation between pre-stretch and external load is obtained. Upon the release of the external load, the silicone tube returns to its initial length. Hence it can be inferred that the tube does not have undergone any permanent deformation rather the change in slope for the curve after 150 mm elongation is due to the effect of reduction in stiffness and thickness.

The effect of pre-stretch on the natural frequency of the empty tube is identified experimentally. The tube length of 36 cm is pre-

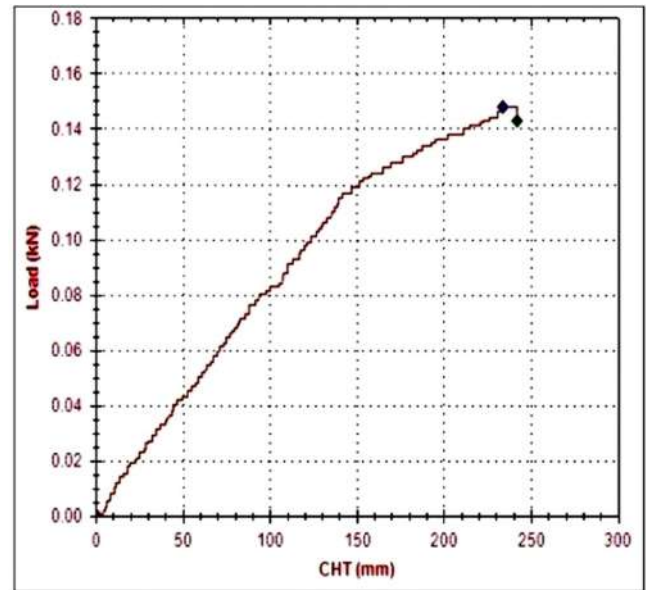


Fig. 3. Load –Deflection curve of Silicone rubber tube.

stretched incrementally and the natural frequencies and damping ratios corresponding to each pre-stretch are found out. The Table 3 shows the first fundamental frequency of the silicone tube under varying pre-stretches. The natural frequency of the tube increases rapidly during initial pre-stretching (till 15% of initial pre-stretch) but later the change in natural frequency upon pre-stretching of the tube is low. This is due to the addition of geometric stiffness of the silicone tube, where the mass of tube remains constant. But latter upon pre-stretching, the thickness of the tube reduces; hence the rate of increase in stiffness also reduces.

### 4.2. Effect of Pre-stretch on damping ratio

The influence of pre-stretch on the damping ratio of silicone tube is studied using a tube of initial length 36 cm, and it is pre-stretched incrementally. The damping ratio and the damping in Hz experienced for every mode of vibrations is obtained from the ME scope software. Fig. 4 presents the impact of pre-stretch on the damping ratio of the silicone tube with and without fluid. The figure reveals that the silicone tube exhibits a decreasing trend for the damping ratio up to a particular pre-stretch. The damping ratio for both empty and fluid-filled tubes was initially high, but it reduces as the tube is pre-stretched further. This trend continues up to 12.5% pre-stretch. The point corresponds to 12.5% pre-stretch can be named as an inflection point for both bare and fluid-filled tubes where the damping ratio has the lowest value. It is found that the addition of pre-stretch from 12.5% of initial length improves the damping of the empty tube as well as the fluid contained tube. Hence it can be found that the damping of the tube, as well as the tube oscillations, can be altered by the application of pre-stretch. The silicone tube upon initial pre-stretching improves the stiffness more vigorously than the latter stage. Hence the damping ratio declines as the tube is pre-stretched initially. But the pre-stretching results in the increase of length and the

Table 3  
Natural frequency of 36 cm long empty tube under different pre-stretches.

Sl no	Length of tube(cm)	First fundamental frequency(Hz)
1	32.4 + 3.6 cm (Pre-stretch)	24.3
2	30.6 + 5.4 cm (Pre-stretch)	29.3
3	28.8 + 7.2 cm (Pre-stretch)	30.1

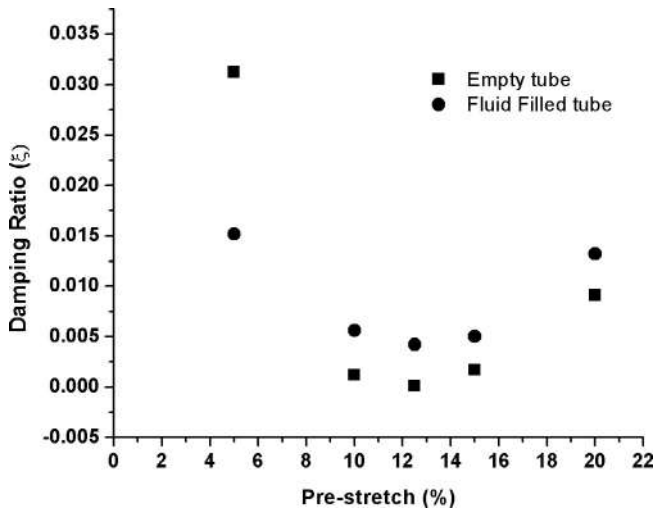


Fig. 4. Effect of pre-stretching on damping ratio.

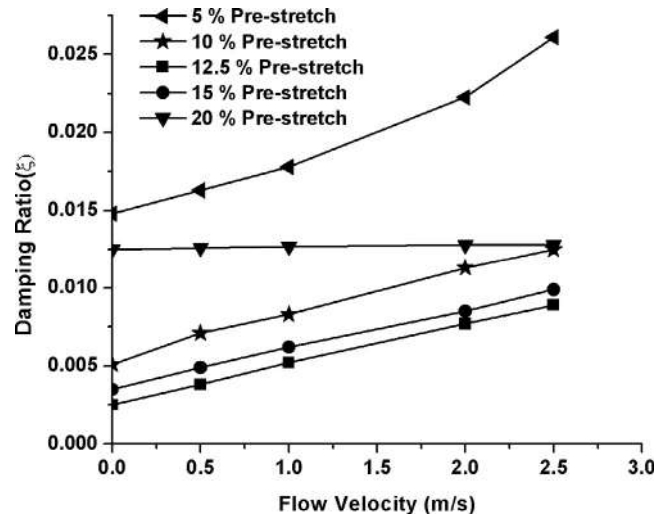


Fig. 5. Effect of flow velocity and pre-stretch on damping ratio.

decrease in the thickness of the tube. Thus the stiffness of tube reduces as the wall thickness drops, and hence it increases tube damping property.

In the unstressed state, elastomers like silicone rubber tube compose cross-linked molecular chains that are highly twisted, kinked and coiled. When a tensile load is applied, these molecular chains partially uncoil and straighten which causes the elongation of the molecular chains in the stress direction [14]. In the current investigation as the tube is stretched to 12.5% of original length due to the uncoiling and straightening of molecular chains, the damping ratio decreases and after this 12.5% pre-stretch an increase in the energy dissipation can take place due to the uncoiling and straightening of cross-linked chains as well as the friction between chains, which explains the reason for increasing trend of damping ratio after this particular pre-stretch of 12.5%.

4.3. Effect of flow velocity on damping ratio

The silicone tube of 36 cm is pre-stretched and a steady-state flow is maintained through the tube from a constant head. For each pre-stretching, the flow velocity is varied from 0 to 2.51 m/s keeping the head over the tube as a constant. The dynamic responses of the tube under different pre-stretch are investigated. Fig. 5 shows the variation of the damping ratio of the tube with flow velocity for different pre-stretches. The general trend is found that the increase in flow velocity improves the damping ratio for a particular pre-stretch value. But for a particular flow velocity as the pre-stretch value increases from 5% to 20% of initial length, the damping ratio is found to be minimum for 12.5% pre-stretch whereas it is maximum for 5% pre-stretch. As discussed in Fig. 3, 12.5% pre-stretch is an inflection point that offers a very low damping ratio. For 12.5% pre-stretched tube, as the flow velocity increases, the damping ratios were found to be less than what obtained for the 10% and 15% pre-stretched tubes. This confirms that the pre-stretching affects the tube wall stiffness vigorously and thereby shows an anomaly in the variation of damping ratio values. The damping ratio exhibits a constant nature over the increasing flow rates for the pre-stretch of 20%. But for pre-stretches from 5% to 15%, the damping ratio shows an increasing trend with increasing flow velocities. The increasing damping ratio corresponding to the increase in flow velocity for low pre-stretched tube is due to the self generated axial as well as circumferential tension in the tube happens due to the perturbations in velocity along radial and axial direction of fluid flow.

4.4. Effect of mass and stiffness proportion on the flow rate

The effect of mass and stiffness proportion on the damping of flexible silicone tube conveying fluid is identified with the help of the Rayleigh damping model. The first and second fundamental frequencies along with the damping ratios (found out experimentally) are used to find the mass and stiffness proportions. The mass and stiffness proportions are evaluated for various pre-stretches and flow rates. Tables 4 and 5 shows the natural frequencies and corresponding damping ratios for fluid-filled tube with zero flow velocity and maximum flow velocity respectively. Fig. 6 shows the influence of mass proportion in pre-stretch and flow velocity. The pre-stretched tube of 5% pre-stretch has a maximum damping ratio. It has been found that the damping ratio increases as the flow velocity increases. This is due to the effect of centrifugal force, which imparts a compression in the tube. This compression actually reduces the natural frequency of the fluid conveying tube and increases damping. The stiffness participation also shows a similar trend (Fig. 7). The increase in geometric stiffness due to pre-stretching increases the damping ratio.

4.5. Effect of flow velocity on natural frequency of tube

Fig. 8 shows the variation of natural frequencies with the flow velocity for various pre-stretching. For lightly pre-stretched tubes, as the fluid flow increases, the tube experiences more compressive

Table 4 First three natural frequencies and damping ratios of flexible tube filled with fluid.

Mode	Natural frequency (Hz)	Damping ratio (%)	Damping (Hz)
Mode 1	12.4	0.518	0.11
Mode 2	36.8	0.639	0.279
Mode 3	79.4	1.21	1.52

Table 5 First three natural frequencies and damping ratios of flexible tube conveying fluid (velocity of flow = 2.5 m/s).

Mode	Natural frequency (Hz)	Damping ratio (%)	Damping (Hz)
Mode 1	12.0	0.577	0.123
Mode 2	36.2	0.853	0.372
Mode 3	78.1	1.36	1.75

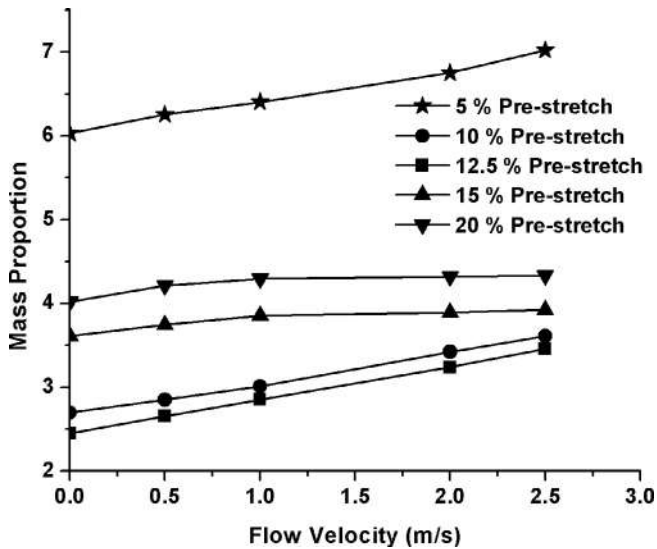


Fig. 6. Influence of mass proportion in damping.

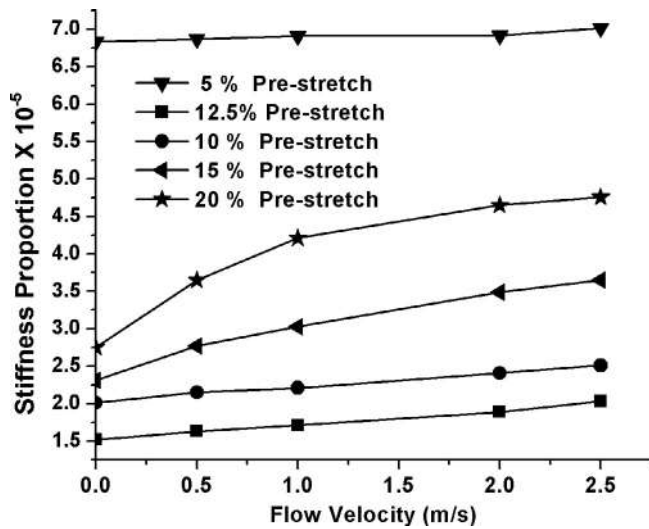


Fig. 7. Influence of stiffness proportion in damping.

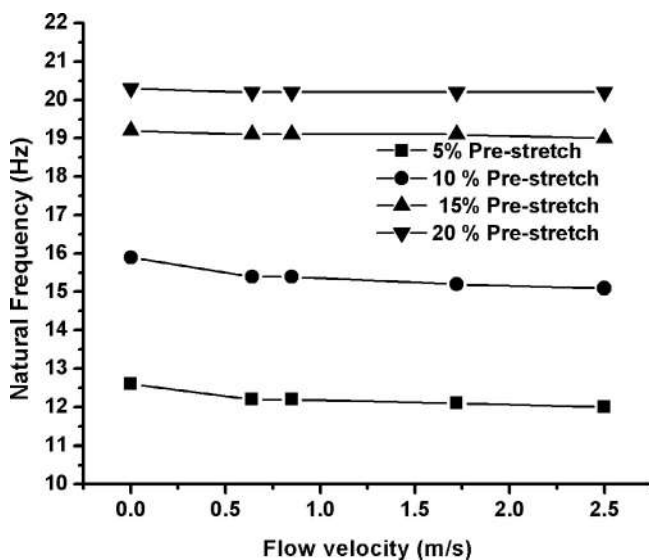


Fig. 8. Effect of flow velocity on the natural frequency of fluid-conveying tube.

load due to the inertia of the fluid. Therefore the natural frequency drops while the flow velocity is increased. But when the tube is pre-stretched further, the geometric stiffness of tube increases which results in a lower natural frequency drop corresponding to the rise in flow velocity.

## 5. Conclusion

In this paper, the experimental investigation on initially stretched silicone tube conveying fluid is reported. The effect of flow velocity and pre-stretch on the vibrational characteristics of the silicone tube is brought out. The Rayleigh linear damping model is used to evaluate the damping characteristics of pre-stretched flexible tubes conveying fluid under variable flow conditions.

The study reveals that pre-stretch has a noticeable effect on the damping ratio for a 36 cm length silicone tube with pre-stretches of 5%, 10%, 12.5% and 15% of initial length. The damping ratio increases as the flow velocity is increased from 0 to 2.51 m/s, but at pre-stretch of 20% of initial length, the values of damping ratio attained a constant value irrespective of change in flow velocity. The stretch rates can affect the area of cross-section which in turn altered the flow velocity to some extent. As the flow velocity increases, the natural frequency was found to be decreasing, but the variations in the natural frequency values at different flow velocities are small.

## CRedit authorship contribution statement

**R. Kamal Krishna:** Conceptualization, Data curation, Investigation, Methodology, Visualization, Writing - original draft. **M Unnikrishnan:** Formal analysis, Investigation, Supervision, Validation, Writing - review & editing. **Jayaraj Kochupillai:** Conceptualization, Data curation, Investigation, Methodology, Supervision, Validation, Writing - review & editing.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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